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*Nuclear Reactor Physics*

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Second Edition

Wiley-VCH

Foreword

This document is meant to be a derivation of various formulae associated with the PN method covered in the NPRE 555 class with Professor Katy Huff in Spring 2018. The topic is covered in Stacey sections 9.6 and 9.7.

Ideas for derivation:

* Equation 9.119
  + 
  + Start with Rodrigues’ formula and the binomial theorem to prove the recursion relation.
* Equation 9.120
  + 
  + Start with Rodrigues’ formula and the binomial theorem to prove the orthogonality relation.
* Equation 9.121
  + 
  + where 

Equation 9.119 is provided,



which is a recursive relationship that is ultimately used in the transport component of the neutron transport equation later in the section.

Consider the Legendre polynomial, , which is a solution to the Legendre differential equation,



The general form of the Legendre polynomial is given by Rodrigues’ formula,



The expression of the polynomial is given by the binomial theorem,



where the expression  is given by,



Next, the derivative must be taken of Eq (4) with respect to . This is done  times. Start with sequential derivatives,







Write this in the general form,



The floor in the upper limit is due to the fact that the binomial theorem is applied to the square of the argument, . Therefore, only on odd derivatives is an additional term lost due to derivatives.

Apply Eq (9) for ,



Simplify,



Next, express the Legendre polynomial as the sum,



The first seven Legendre polynomials are verified in Appendix A.

Substitute Eq (12) into Eq (1) to verify the identity,





Change the summation index on the second term,



Simplify,



Change factorials and both the sums on the right-hand side,



Simplify,



Simplify,



Factor the numerator,



Simplify,



Combine the two summations on the right-hand side,



The equality holds for all  except the last entry. There are two cases to investigate:  even and  odd. If  is even, then the final term should add an additional term identical to the other terms. If  is odd, the final term should be zero as this is an additional term not present on the left-hand side.

 even:

If  is even,



Therefore,



Simplify,



Simplify further,



Further,



Further still,



Note that



Apply this,



QED

Next, consider the odd case,



Apply this,



Simplify,



Simplify,



Simplify,



Simplify,



Finally, note that



for odd . The final expression for odd  is therefore,



QED

Use Eq (12) to verify the first seven Legendre polynomials,

General form,

















































































